IOSR Journal of Electronics and Communication Engineering (IOSR-JECE)
e-ISSN: 2278-2834,p- ISSN: 2278-8735.Volume 11, Issue 6, Ver. II (Nov.-Dec .2016), PP 01-11
www.iosrjournals.org

Top Efficiency on Communication Theory

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Abstract: *Nowadays we know that at least for discrete systems there is an equivalence between Time Reversal and the use of meta materials for the so called overcoming of the diffraction limit. But it remains a question concerning the efficiency on transporting information. In this paper, we make a review of some previous works to the end of make a fusion between the uses of any of these two equivalent phenomena and two new fields namely the Plasma Sandwich Model, and the association of a new class of electromagnetic resonances and left-hand-materials conditions. On the aim to obtain a rule that guides us for obtaining an observable parameter we recall some of our results concerning communication theory and we define a measure of the loss of information when left-hand materials conditions domains a broadcasting process. We then obtain a more general recipe that can be applied also for understanding left-hand material properties and other related physical systems. Because on the theory appears naturally the Green functions, we give a special place for the retarded and advanced versions of them. As an extremely important result we show how we can link the concepts of recording time and resonances to give a singular algorithm to push the efficiency to the top. Also we give a simple academic example and show how this operates.*

Keywords: *Communication theory, Time reversal, Resonances on a channel, Left-hand materials.*

I. Introduction

Currently, the new ways of transmitting information are pushing Communications to extreme challenges. We have noted that it is well known, that random scattering of microwave or radio signals may enhance the amount of information that can be transmitted over a channel. This is because the phase space where the transmitting phenomena occur grows with every single collision of the initial signals because the scattered ones reach another phase space regions and any new region provides additional information. But Communications involves not only anelectromagnetic waves issue and indeed we have an equivalent behavior in acoustics.The meaning of the sentence"overcoming of the diffraction limit"is maybe a point of viewof physical phenomena in which we can observe an unusual concentration of a signal but the only very important think is the amazing of that localization. Whatever we know that the tools emerged from different works have increased the ways to improve the signals focusing and to avoid the loose of information.

Time reversal, or phase conjugation in the frequency domain, is a process where a source at one location transmits sound or electromagnetic waves, which are received at another location, time reversed (or phase conjugated), and retransmitted. The retransmitted signals then focuses back at the original source location, where the reception is relatively free of multipath contamination.

If we compared with the free space resolution, the multiple scattering of the obstacles enlarged the effective aperture in the so-called Time Reversed Mirror for acoustic signals provided they are placed in aleatory manner. The focal properties of the time-reversal process were able to suppress cross talk even though the receivers were at the same range and only separated by depth.

In this work we recall some of our results and explore how information theory can help us to define a measure of the lack of information in Time Reversal Techniques no matter if we talk of electromagnetic nor acoustic waves, but now we add a very innovating chapter on Communications that is the role of the new models of resonating broadcasting[1-12]. And even when technically it is not possible to measure different recording times for electromagnetic waves we can alternatively uses those times as very important parameters. Indeed we introduce the recording time explicitly in the advanced Green function.

We assume that our lack of information measure is also validfor understanding left-hand materials properties[13-18].

II. Left and right hand materials

Materials existing in nature have a positive refraction index. These are called Right Hand Materials (RHM) in order to distinguish them from artificially created materials which have a negative refraction index, and which are called Left Hand Materials (LHM) [13-18]. The complex refraction index *n* is defined as the quotient of the velocities of an electromagnetic wave in the medium and in empty space. This refraction index can be written in terms of the magnetic permeability μ and electric permittivity ε as $n^2 = \mu\varepsilon$, when both μ and ε are negative for a range of frequencies we can write the permeability as $\mu = i^2 |\mu|$ and the permittivity as $\varepsilon = i^2 |\varepsilon|$ and then we can see that trivially $n < 0$. Among many other consequences, this means that group and phase velocities are opposite.

The extraordinary property of LHM of allowing waves to travel in opposite sense to energy transmission translates itself in the fact that the so-called evanescent waves emitted by observed objects grow inside the material instead of diminish. At the end of their journey the electromagnetic waves will have recovered the information which otherwise is normally wasted.

Among meta materials we find lenses made up of thin films of some metallic compounds. In particular, an ultrathin film of silver placed between two materials of positive refraction index constitutes a Superlens with a negative refraction index *n*. The reported experimental results are amazing, among other reasons, because there are applied to images in the visible spectrum.

In this case, while evanescent waves are losing amplitude as they go through a "right hand" medium (with a positive refraction index), when they reach the film they grow until, when the image is formed, they superpose with the field that usually forms the image, enhancing it and giving it a higher definition beyond the diffraction limit. The image has a high resolution of λ/6 (λ is the wavelength used for illumination). As we will see in chapters IV and V we can see that the transmitting media in a broadcasting process also behaves like a left-hand material because of a plasma-sandwich model are working.

III. Time reversal techniques and the overcoming of diffraction limit

As we said above it is a matter of difference in opinion, if we call or not in optics, overcoming the diffraction limit to the increase of focusing on a spot more little than the wavelength but indeed this physical phenomenon requires a negative refraction index or a super-resolution due to a fortunately combination of individual waves.On the other hand, in acoustics this possibilityis associated with time reversibility. Indeed we have shown recently also an electromagnetic version of TRT[18, 22, 27]. So we have two different kinds of systems that share the same mathematical description, so with the aim of describe both of them simultaneously for their use on Communications, we start with a discrete acoustical system and then go to a general treatment. Due to the fact that the formalism we have developed for the study of time reversibility refers to discrete systems, we will adhere to this type of systems, without losing generality. Firstly, let us recall that the wave equation can be written as

$$k\left(\bar{r}\right)\frac{\partial^2 u\left(\bar{r},t\right)}{\partial t^2} = \nabla^2\left(u\left(\bar{r},t\right)/\rho\left(\bar{r}\right)\right) \quad (1)$$

$\rho\left(\bar{r}\right)$ being the mass density and $k\left(\bar{r}\right)$ the compressibility of the propagation medium, while $u\left(\bar{r},t\right)$ is the acoustic signal that for a discrete system may be written as $u\left(\bar{r}_j,t\right) = u_j(t)$, where $\bar{r}_j$ may be the position of a transducer or a dispersion site, a source or a sink.

Acoustic time reversibility has its foundations on the fact that the wave equation is second order in time, which allows solutions which travel toward the future or the past, as if a film was ran forwards or backwards. One of the conditions to carry out time reversal successfully is that the system be ergodic, which guarantees that the signal may travel both senses in time. To overcome the diffraction limit we must describe the signal propagation toward the future or past by means of equations of the same type [18, 22, 27]. Considering a signal travelling towards the past, the corresponding integral equation is

Where $G^{(\circ)*}\left(\bar{r}_j,T-t';\bar{r}_s,t\right)$ is the free Green function, A_s^* are the complex dispersion

$$u_s(T-t) = u_s^{(\circ)}\left(T-t\right) + \sum_j \int_{-\infty}^{\infty} A_s^* G^{(\circ)*}\left(\bar{r}_j,T-t';\bar{r}_s,t\right) u_j\left(T-t'\right)dt' \quad (2)$$

coefficients and $u_s(T-t)$ is the returning signal that has travelled toward the past. The term $u_s^{(\circ)}\left(T-t\right)$ is a sink term, which guarantees that the outgoing and returning equations are both inhomogeneous integral equations. In this equation there is a parameter T, which represents the time during which the outgoing signal (the one travelling toward the future) is being considered (a recording during a time T might have been carried out). It is observed[9] that the time-reversed signal has a definition of a fourteenth of λ, the wavelength of the used signal for acoustic signals but also for electromagnetic waves.

IV. The Vector Matrix Formalism

Now, making a leap to electromagnetic waves we recall the vector matrix formalism which nearly reproduce or better generalize the discrete scalar time reversal acoustic model and include a new model of behavior on discrete broadcasting systems that is the one we called the Plasma Sandwich Model (PSM) and we put some associated parameters appeared on it into the namedVector Matrix Formalism (VMF)[8, 18, 22]. But we must underline that is the resonant behavior the one we must be taken into account for an extraordinary resolution. To this end, we remember that a wave equation like equation (1) can be written as the Fourier transform of an integralHomogeneous Fredholm Equation (GHFE) for resonances, and doesn't matter if for acoustic or electromagnetic ones. To analyze the resonant behavior we have eliminated the source term (for this reason we showfirst the homogeneous equation) and obtain the following equation:

$$\left[\mathbf{1}-\eta_R(\omega)\mathbf{K}^{(\circ)}(\omega)\right]_n^m \overline{w}_R^n(\omega)=0 \qquad (5)$$

Where the kernel $\mathbf{K}^{(\circ)}(\omega)$ is the product of the free Green function $\mathbf{G}^{(\circ)}(\omega)$ with the interaction $\mathbf{A}$ so explicitly

$$\left[\mathbf{1}-\eta_R(\omega)\mathbf{G}^{(\circ)}(\omega)\mathbf{A}\right]_n^m \overline{w}_R^n(\omega)=0 \qquad (6)$$

It is not difficult to think that if we have a transfer matrix description of a problem we must have a VMF version of it. Of course there are very important differences between these formalisms, for example, VMF includes explicitly a mechanism to makes easy a time reversal process. Also we have a frequency domain instead a time dependent one, the former the appropriate for information theory applications. But possibly the most important difference is that VMF formalism includes the concept of the resonant solutions. In the present work, we start with the appropriate VMF generic version of the PSM and then introduce the relevant parameters in the next section.

V. The Kernel with PSM parameters

In this section we find the resonant frequencies for a specific problem, but we must remember that those resonant frequencies can be used only to associate an interval of frequencies of a real signal to a device that could be an antenna. The form of the kernel depends of the response of the media in some circumstances that can vary even from a different time interval. So we use an example that is very easy to work but that is not important how is the shape of the signal we used to get it. Now, we can find the resonant frequencies in this academic example. To this end we choose a convenient kernel $\mathbf{K}^{(\circ)}(\omega)$, for simplicity we do not take into accounts the three components of the electromagnetic field. Supposing we only has one, but we have two emitting antennas. A possible kernel is [1, 3-7]:

$$\mathbf{K}^{(\circ)}(\omega)=\begin{pmatrix} \dfrac{\sin(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta} & -i\dfrac{\cos(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta} \\ i\dfrac{\cos(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta} & \dfrac{\sin(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta} \end{pmatrix} \qquad (7)$$

In equation (7) we have introduced the Plasma Sandwich Model (PSM) parameter d. We are used the definition:

$$\delta=\kappa\overline{d}_M \qquad (8)$$

In definition (8) κ has the physical meaning of the wave number of an incident beam that interacts with the magnetic and electric fields in a way that the whole kernel is the expressed in equation (7). But $\overline{d}_M$ is an average thickness of a plasma-magnetized layer that generates this interaction. The parameter w_p is an average value for the plasma frequency in the same magnetized plasma layer that can be expressed in terms of the local electron concentration in the layer as:

$$\omega_p=\frac{1}{2\pi}\left(\frac{Ne^2}{m\varepsilon_0}\right)^{\frac{1}{2}} \qquad (9)$$

In this equation N is the electron concentration, ε_0 is the permittivity of vacuum and e the electronic charge.

We can observe that the change in these parameters gives different broadcasting regimes[5]. The PSM also supposes that we have not a stationary and unique set of iterated layers but a series of sets changing with time and therefore with different effects for distinct frequencies. At this point, it is important to remember that the equation we must solve is equation (5) where

$$\mathbf{K}_{i,j,m}^{n(\circ)}(\omega)=\left\{\begin{matrix} 0 \ \ if \ i=j \\ A_j^{n,m}G_{\omega}^{n,m(\circ)}(\overline{r_i},\overline{r_j}) \ if \ i\neq j \end{matrix}\right\} \qquad (10)$$

The conditions for resonances are that Fredholm's determinant for the equation (7) equals zero, and that Fredholm´s eigenvalue λ equals to one [6, 9, 20, 21].

These two conditions give us the resonant frequencies for the system constituted by these two antennas but dependent on the plasma sandwich model parameters. Now, we must remember that resonances have a special behavior that can be represented by a complex frequency:

$$\omega = \mathrm{K} - i\Lambda \qquad (11)$$

The imaginary part Λ is responsible for the amazing transformation of the evanescent waves for travelling ones. Also we have the relation between ω and the wave number κ, that is

$$\kappa=\sqrt{\mu\varepsilon}\,\omega \qquad (12)$$

By substituting (11) and (12) into equation (7) we have that resonance condition can be written as:

$$\Delta\begin{pmatrix} A & -B \\ B & A \end{pmatrix}=0 \qquad (13)$$

Where

$$A=\rho_p\left(\sin(\rho_p)\mathrm{ch}\left(\gamma_p\right)-\lambda_p\right)+\gamma_p sh\left(\gamma_p\right)\cos\left(\rho_p\right)+ i\left(\rho_p sh\left(\gamma_p\right)\cos\left(\rho_p\right)+\gamma_p\lambda_p\right) \qquad (14)$$

And

$$B=\gamma_p\cos\left(\rho_p\right)ch\left(\gamma_p\right)+\rho_p\sin\left(\rho_p\right)sh\left(\gamma_p\right)+ i\left(\rho_p\cos\left(\rho_p\right)ch\left(\gamma_p\right)-\gamma_p\sin\left(\rho_p\right)sh\left(\gamma_p\right)\right) \qquad (15)$$

In equations (14) and (15) we have used the following definitions:

$$\sigma_M=\overline{d}_M\sqrt{\mu\varepsilon} \qquad (16)$$

$$\rho_p=\sigma_M\left(\mathrm{K}^2-\Lambda^2-\omega_p\mathrm{K}\right) \qquad (17)$$

$$\gamma_p=\sigma_M\Lambda\left(\omega_p-2\mathrm{K}\right) \qquad (18)$$

$$\lambda_p=\lambda\left(\rho^2{}_p+\gamma^2{}_p\right) \qquad (19)$$

To have an image of the solutions of equation (13) we can make $\mathrm{K}=x$ and $\Lambda=y$ those are the real and imaginary parts of ω, and fix the value for the plasma frequency ω_p so we have the following image:

We obtain for the particular conditions:

$$\mathrm{K}=\Lambda \qquad (20)$$

$$\omega_p=10^6\,Hz \qquad (21)$$

The solutions (resonances):

$$x_1=5.009\times10^5\,\mathbf{Hz} \qquad (22)$$

$$x_2=-985.99\mathbf{Hz} \qquad (23)$$

In this case only x_1 is properly a resonance and x_2 has not physical meaning but maintain their orthogonality properties.

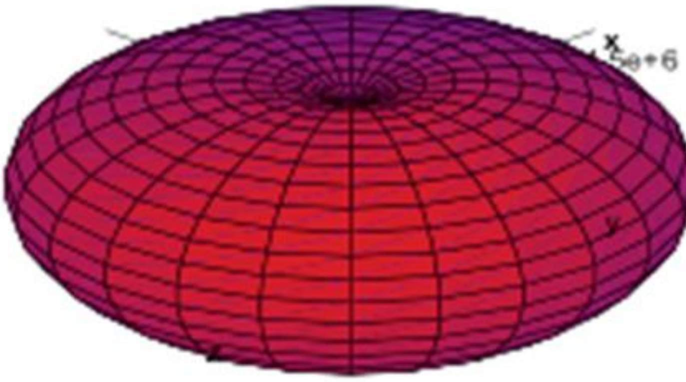

Figure 1. Image of the solutions of equation (13) when the related equation is $987.93(x^2-y^2-10^6) = y\,(10^6-2x)$.

VI. Communication theory measurement of information loss

Because we have now a wide vision of the loss of information and we know that this is the reason that the images are not perfect, we can use the results [23-27] of E. Shannon, H. Nyquist, N. Wiener, R.V.L. Hartley, E. Hopf, and other authors that have formulated a measure of the loss of information in communication systems. We support our mathematical results on related works [6, 9, 18, 20, 22], whichgive us a solid theoretical frame to our present and future papers . Indeed because the *capacity*of a channel and *entropy* are very close concepts, we can use some of the results we have cited above to answer the problem for TRT and LHM.
Basically, we recall two theorems:

Theorem I.
If the signal and noise are independent and the received signal is the sum of the transmitted signal and the noise then the rate of transmission is

$$R = H(y) - H(n) \tag{12}$$

This means that the rate of transmission is the entropy of the received signal less the entropy of the noise. The channel capacity is

$$C = \underset{P(x)}{Max} H(y) - H(n) \tag{13}$$

Theorem II.
The capacity of a channel of band Wperturbed by white thermal noise power N when the average transmitter power is limited to P is given by
IV.

$$C = W \log \frac{P+N}{N} \tag{14}$$

In this expression P is the average power of the transmitted signal and N the average noise power.
From these two theorems we make our proposal for a channel where we have lost information in three ways. That is, we have limitations on the maximum frequency W (band), the presence of different classes of noise, and on a limited time T available for a time reversal process. Then defining a joint average for the power $Q(n,T)$ the channel capacity is

$$C_T = W \log \frac{P + Q(n,T)}{Q(n,T)} \tag{15}$$

Thisremainsequal to zero when $P = 0$. The very significant feature of this proposal is the explicit dependence on T , in both the joint average power and the channel capacity, as opposed to the conventional treatment of the signal time duration that is considered as a limit process which tends to infinity. This is a consequence of the explicit form of the Fourier transform of the time reversed Green function that changes with a factor $e^{i\omega T}$, so even if we are not forced to do so, we can think of it as a parameter which defines the channel. We can think of an arbitrary channel but, when we use it to reverse any signal in time we follow a different process depending on the time *T*we decide to fit. Then we can label the channel with each T as a different one and of course with a different capacity with those corresponding to other values of T. Because of the arguments expressed previously in this work, we can use this measure to the same extent on LHM, ATR and TRT. For a related discussion of the equivalence of the time reversal methods and the employment of left hand materials we can see reference [28], and for the use of time reversal on antennas we can see also reference[14].

VII. An academic example

In order to give an insight into information measurement applied to TR, let's propose that our system behaves like a filter (so in this particular example we have not loss if we select $t < T$), in addition we also propose that instead of the pulse in Eq. (13) we have another form like [10]

$$\frac{\sin(2\pi Wt)}{2\pi Wt} \tag{16}$$

And also that we have instead of the incoming signal in Eq. (15) another like [10]

$$\frac{1}{2}\frac{\sin^2(\pi Wt)}{(\pi Wt)^2} \tag{17}$$

The input function Eq. (16) is a sample of a more general function generated by the sum of a series of shifted functions

$$a\frac{\sin(2\pi Wt)}{2\pi Wt}. \tag{18}$$

Where a , the amplitude of the sample is no greater than $\sqrt{S}$. (S is the peak allowed transmitter power).
The *channel capacity* would be [23]approximately

$$C_T = W\log\left(\frac{S+Q(n,T)}{Q(n,T)}\right) \tag{19}$$

This occurs if it is provided that *S/N* is small.
In the time reversal process we have shown that for each Fourier component we should add a complex exponential factor dependent on T .But we know now that the tool is the same and that only the numerical value of channel capacity C_T changes. We see how in practice the time reversal parameter T appears explicitly but also that when we cut the time duration of reversed signal it ispossible to consider them as an additive contribution to $Q(n,T)$.
But the form of equation (19) suggests a generalized measure of a blend or mix channels capacity when sharing the same band *W* and differ only by the recording times $T_1, T_2, \cdots, T_n$

$$C_{T_1,T_2,\cdots,T_n} = W\log\left(\frac{S+Q(n,T_1,T_2,\cdots,T_n)}{Q(n,T_1,T_2,\cdots,T_n)}\right) \tag{20}$$

$T_i = 0$. The fact that we are using the same band but different cutting limits $T_1, T_2, \cdots, T_n$, also suggests that we can design an appropriate filter that can distinguish between signals according to the recording time that is we can superpose signals with the same frequency range but with different recording times. In a previous work we have sketched a filter, but now we give a better-defined device so we propose (see Figure 2) as a hint to get the filter, the following steps for both the transmitter and the receiver:

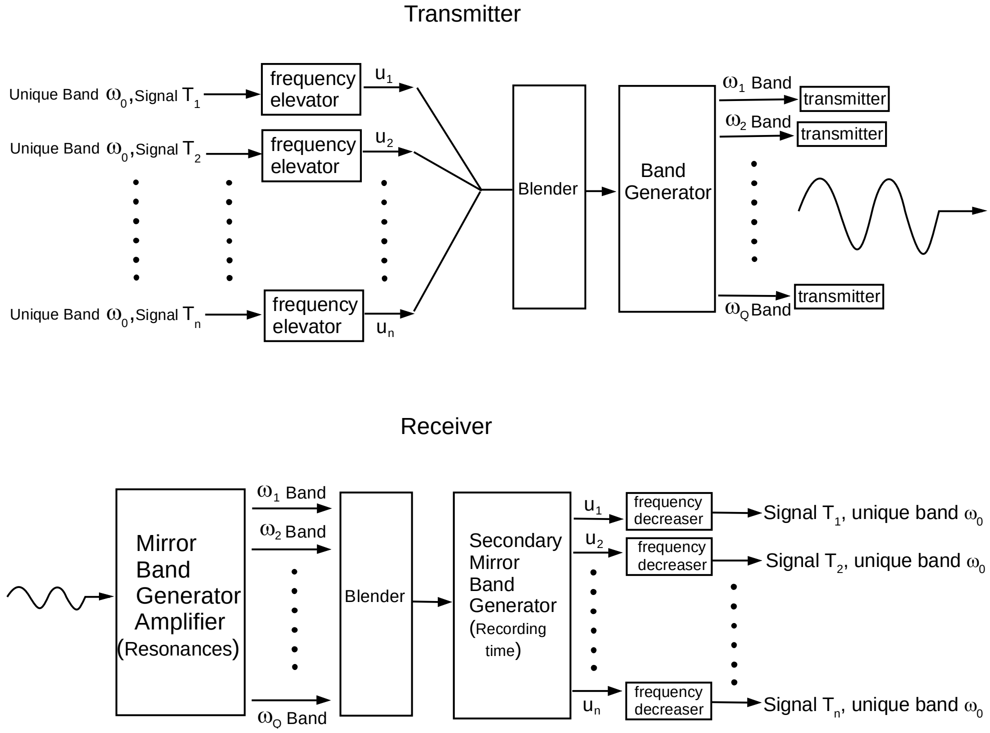

Figure 2. We show the flow chart for a proposal device. This can emit andread the blended messages with recording times $T_1, T_2, \cdots, T_n$ beneath to the unique band ω_0.

Transmitter

First increase the n frequencies onthe unique entrance band $B(\omega_0)$(that is centered in frequency ω_0)incoming from the inverse of $T_1, T_2, \cdots, T_n$, then the n new central frequencies $u_1, u_2, \cdots, u_n$(and their signals) enter a blender and the mixed signal is taken by a Band Generator and separated in Q new bands centered at the frequencies $\omega_1, \omega_2, \cdots, \omega_Q$(each corresponding with a resonant frequency). Finally, each band enters his signal transmitter.

Receiver

The travelling Q signals enter the Mirror Band Amplifier, so called because it knows that there are Q resonant frequencies and then can create (or separate the signal in Q resonant bands)Q sub-bands and amplifythe signal in each band (at this moment each band carry a piece of the originals n different signals) after this, the Q signals are blended and then sending to a Secondary Mirror Band Generator which knows that there are n recording times $T_1, T_2, \cdots, T_n$ and because of that it can create n bands with the higher central frequencies $u_1, u_2, \cdots, u_n$ (these last signals could be amplitude modulated signals) and distribute the blended signal among them. Then every signal on each band enters a frequency decreaser (the inverse operation performed by the frequency elevators in the transmitter) so we retrieve the n original signals corresponding to the unique band $B(\omega_0)$.

For the example on section **V** we have that the total number of resonances is $Q = 2$, and the two resonant frequencies are $\omega_1 = 5.009 \times 10^5$ **Hz** and $\omega_2 = -985.99$ **Hz**

VIII. Left-hand materials conditions and a Little Theorem

Based on the equivalence of the TRT and the properties of the Green function, we can trust that any discussion about the interaction of meta materials with electromagnetic field can be done through this function and simultaneously observe the effect of a time reversal. For this reason we can now describe the error in terms of the Green function by the hypothesis that LHM can be put to test by forward and backward in time signals and read the results with two points of view: first the direct effect of the loss of information because of the limited record time *T* or second, how the negative refraction index helps to preserve information. Now we can review our previous results and generalize by the use of the kernels, so we cancharacterize the capacity of a channel in many different circumstances. So we have made use of the analogies [28]between the TRT and the employment of LHM in order to propose that we can express the capacity of any of these negative refraction index materials in the same terms or procedures as those of TRT. Also we can propose an identical description for the channel capacity that is Eq. (15) and its generalization equation (20). Then, the matrix formalism for discrete systems can be used to characterize the channel capacity of transmission of information in a process of time reversibility using the Fourier transforms of the Green functions (properly we use the kernels with the interaction matrix $\mathbf{A}=\mathbf{1}$) forward and backward. That is, if by a first step the signal transforms like (in the following equations I and F stands for initial and final places)

$$\overline{Y}_F(\omega)=\left[\mathbf{1}+\mathbf{R}(\omega)\right]\overline{X}_I(\omega) \tag{21}$$

And then in a second step returns to the initial place by means of the operation

$$\overline{Z}_I(\omega)=\left[\mathbf{1}-\mathbf{K}^{(\circ)}(\omega)\right]\overline{Y}_F(\omega) \tag{22}$$

Then the complete signal trip would be

$$\overline{Z}_I(\omega)=\left[\mathbf{1}-\mathbf{K}^{(\circ)}(\omega)\right]\left[\mathbf{1}+\mathbf{R}(\omega)\right]\overline{X}_I(\omega) \tag{23}$$

So that by defining the error in the time reversing process by

$$\delta\overline{X}_I=\overline{X}_I-\overline{Z}_I \tag{24}$$

We can write this like

$$\delta\overline{X}_I(\omega)=\overline{X}_I-\left[\mathbf{1}-\mathbf{K}^{(\circ)}(\omega)\right]\left[\mathbf{1}+\mathbf{R}(\omega)\right]\overline{X}_I(\omega) \tag{25}$$

Or

$$\delta\overline{X}_I(\omega)=-\left[\mathbf{R}(\omega)-\mathbf{K}^{(\circ)}(\omega)-\mathbf{K}^{(\circ)}(\omega)\mathbf{R}(\omega)\right]\overline{X}_I(\omega) \tag{26}$$

Equation (26) is a corollary that shows explicitly the role of both the forward and backward Fourier transform of the Green function (we have done $\mathbf{A}=\mathbf{1}$on equation (6) for convenience and also for the complete kernels $\mathbf{K}(\omega)$and $\mathbf{R}(\omega)$).Equation (26) is very clear about the origin of the errors because we can see for example that in the case that the forward and backward Green functions are mathematically one the transpose conjugated of the other for a perfect time reversal (when acting the first on a column vector and on a row vector the other), we get that the error is zero and that the error increases as the differences of both functions also increases. In a very special case, we can then propose that $\mathbf{K}(\omega)$and $\mathbf{R}(\omega)$only differ by the factor $e^{i\omega T}$ or $e^{2\pi i\frac{\omega}{\omega_T}}$ when the only source of error is the recording time *T*, so that we obtain from equation (25) that:

$$\delta\overline{X}_I(\omega)=-\left[e^{-2\pi i\frac{\omega}{\omega_T}}\mathbf{K}(\omega)-\mathbf{K}^{(\circ)}(\omega)-\mathbf{K}^{(\circ)}(\omega)e^{-i\frac{\omega}{\omega_T}}\mathbf{K}(\omega)\right]\overline{X}_I(\omega) \tag{27}$$

In equation (27) the function $e^{-2\pi i\frac{\omega}{\omega_T}}\mathbf{K}(\omega)$has the form of the Fourier transform of the Green function but with the argument translated by an amount equal to the recording time *T* that appears explicitly in equation (19) that is the Fourier transform of:

$$\mathbf{K}(t-T) \tag{28}$$

But with the time running backward; so as we will show in a moment, if *T* is very short, the error will be very huge. On the contrary, if the time goes to infinity the error will go to zero. Resuming, the new equations (15),

(19), (20), (25), (26) and (27), make possible a characterization of the lost information in left-hand materials not only for microwaves range, but also for visible frequencies because we have extended recently the time reversal techniques (see reference [3, 10]).
Now, we can define:

$$\mathcal{K}=e^{-2\pi i\frac{\omega}{\omega_{T}}}\mathbf{K}$$

(29) So we can write equation (27) like:

$$\delta\overline{X}_{I}(\omega)=-\left[\mathcal{K}(\omega)-\mathbf{K}^{(\circ)}(\omega)-\mathbf{K}^{(\circ)}(\omega)\mathcal{K}(\omega)\right]\overline{X}_{I}(\omega) \quad (30)$$

And because the kernel of theFourier transform of the Generalized Inhomogeneous Fredholm Equations (GIFE) satisfies the following integral equations:

$$\mathcal{K}=\mathbf{K}^{(\circ)}+\mathbf{K}^{(\circ)}\mathcal{K} \quad (31)$$

$$\mathcal{K}=\mathcal{K}^{(\circ)}+\mathcal{K}^{(\circ)}\mathcal{K} \quad (32)$$

While equation (31) exactly represents the problem with a finite recording time T , equation (32) represents a hypothetical problem in which the recording time is infinite.
Substituting equation (31) into equation (30) we have:

$$\delta\overline{X}_{I}(\omega)=-\left[\mathcal{K}(\omega)-\mathcal{K}(\omega)\right]\overline{X}_{I}(\omega) \quad (33)$$

Then we can suppose that the two kernels in equation (33) represents the real and the hypothetical problem described above. Of course we see that if real conditions approximates the ideal ones, the error is clearly zero. But we can factorize the interaction matrix in equation (33):

$$\delta\overline{X}_{I}(\omega)=-\mathbf{A}\left[\mathcal{G}(\omega)-\mathcal{G}(\omega)\right]\overline{X}_{I}(\omega) \quad (34)$$

But equation (34) says clearly that the error does not depend on the form of the interaction, only depends on the recording time T . Even we have supposed that the only source of error was the recording time, we does not suppose any particular behavior for the interaction. So we have proved a little theorem:

Theorem III.
In the time reversal problem and for left-hand materials conditions the normalized error:

$$\frac{\delta\overline{\mathrm{X}}_{I}(\omega)}{\left\|\mathbf{A}\right\|} \quad (35)$$

Is independent of the explicit form of the interaction provided the last is isotropic ($\mathbf{A}^{-1}=\mathbf{A}^{t*}$).

Returning to the time representation, for the time dependent retarded isotropic free Green function related to $\mathbf{K}^{(\circ)}(\omega)$ we has explicitly

$$G^{m,n(\circ)+}(\overline{r}_{j},t;\overline{r}_{j},t')=G^{(+)}(\overline{r}_{j},t;\overline{r}_{j},t')=$$

$$\frac{d\left(t'-\left[t-\frac{\left|\overline{r}-\overline{r}'\right|}{c}\right]\right)}{\left|\overline{r}-\overline{r}'\right|} \quad (36)$$

And for the advanced time dependent free Green function related to $\mathbf{R}^{(\circ)}(\omega)$

$$G^{m,n(\circ)-}(\bar{r}_j,t;\bar{r}_j,t') = G^{(-)}(\bar{r}_j,t;\bar{r}_j,t') =$$

$$\frac{d\left(t' - \left[t + \dfrac{\left|\bar{r} - \bar{r}'\right|}{c} - T \right] \right)}{\left|\bar{r} - \bar{r}'\right|} \qquad (37)$$

That is the recording time appears explicitly in the advanced Green function and we can show that its value makes possible to blend many signals on the same channel without interference. It is important to note that for resonances the relevant Green functions are precisely the free ones and not the complete ones as we can see on equations (5) and (6).

IX. Conclusions

We have shown in equations (15), (19), (20), (25), (26) and (27), that it is possible to use the equivalence between TRT techniques and the properties of the Green function to define the capacity of a channel, the loss of information and the error in the time reversal process. But also that we can extend our results to describe the performance of LHM when interact with electromagnetic field forward or backward in time. Then, because of both the interpretation of a resonance with left-hand materials conditions and the PSM, we designed a broadcasting system that is capable for distinguishes between signals according to their recording time, and that we can superpose signals with the same frequency range but with different recording times with a little loss because of resonance technology, then we are pushing Communications efficiency to the top. In addition we have proved that for the TRT and LHM the normalized error is independent of the particular behavior of the interaction.

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